

Department of Physics, IIT Delhi

II Semester 2007-2008: PHL110 Fields and waves I Minor
 Time: 1 Hour Answer all questions Maximum Marks:25

1. (a) Compute the divergence of the function

$$\vec{v} = (r \cos \theta)\hat{r} + (r \sin \theta)\hat{\theta} + (r \sin \theta \cos \phi)\hat{\phi}.$$

Verify the divergence theorem for this function, using as your volume, a hemispherical bowl of radius R with its base in the $x y$ plane, and centred at the origin

(b) Evaluate the integral

$$I = \int_V e^{-r} \left(\vec{\nabla} \cdot \frac{\vec{r}}{r^2} \right) d\tau.$$

Here V is a sphere of radius R centred at the origin

2. A non-conducting annular ring with uniform surface charge density σ on the upper surface, has inner radius a and outer radius b . It lies in the $x y$ plane with centre at the origin. Compute the potential and the electric field due to this at any point on the z axis, (say $z = 2a$). 5.

$$\int (\nabla \cdot \vec{v}) d\tau = \int_S \vec{v} \cdot d\vec{a}$$

3. A spherical conductor of radius a carries a charge Q . It is surrounded by a linear dielectric material of susceptibility χ_e up to a radius b . Find the electrostatic energy of this configuration. 6.

$$E = \frac{\sigma a}{2\epsilon_0} \left(\frac{1}{\sqrt{a^2+z^2}} - \frac{1}{\sqrt{b^2+z^2}} \right)$$

$$P = \frac{\sigma}{\epsilon_0} \left(\frac{\sqrt{a^2+z^2}}{-\sqrt{a^2+z^2}} \right)$$

$$U = \frac{b-a}{8\pi\epsilon_0} (1+\chi_e)^3 Q^2$$

4. Give short reasons as to whether the following statements are true or false. (No reasons = no marks)

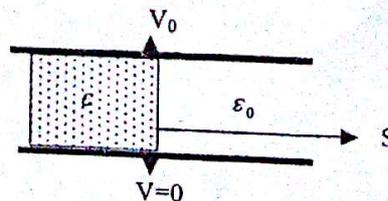
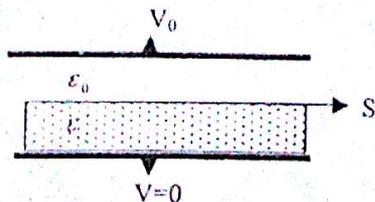
(a) Out of all the radial fields (of the form $\vec{X} = f(r)\hat{r}$), only the inverse square law field has the property that its total flux (except at the origin) is conserved. 2.

(b) A conducting sphere of radius R has two non-overlapping cavities of radius $R/4$. It is connected to a voltage source V . A charge $+Q$ is now placed at the centre of one of the cavities. The potential in the other cavity will be unchanged. 2.

(c) There is a nonzero bound surface charge density at the boundary S between the dielectric and vacuum in both the half filled infinite parallel plate capacitors, shown below. 2.

PLEASE RETURN THE FORMULA SHEET AFTER THE MINOR.

$$\epsilon = \epsilon_0(1 + \chi_e) = \epsilon_0\epsilon_r$$



Handwritten notes: \vec{E} , \vec{F} , \vec{J}

