Department of Physics, IIT Delhi

II Semester 2007-2008: PHL110 Fields and waves I Minor

Time: 1 Hour Answer all questions Maximum Marks:25

1 (a)Compute the divergence of the function

$$\vec{v} = (r\cos\theta)\hat{r} + (r\sin\theta)\hat{\theta} + (r\sin\theta\cos\phi)\hat{\phi}.$$

Verify the divergence theorem for this function, using as your volume, a hemispherical bowl of radius R with its base in the x y plane, and centred at the origin

(b) Evaluate the integral

$$I = \int_V e^{-r}\left(\vec{\nabla}\cdot\frac{\vec{r}}{r^2}\right)d\tau.$$

Here V is a sphere of radius R centred at the origin 8.

2 A non-conducting annular ring with uniform surface charge density σ on the upper surface, has inner radius a and outer radius b. It lies in the x y plane with centre at the origin. Compute the potential and the electric field due to this at any point on the z axis, (say $z = 2a$). 5.

3. A spherical conductor of radius a carries a charge Q. It is surrounded by a linear dielectric material of susceptibility χ_e up to a radius b. Find the electrostatic energy of this configuration. 6.

4 Give short reasons as to whether the following statements are true or false. (No reasons = no marks)

(a) Out of all the radial fields (of the form $\vec{X} = f(r)\hat{r}$), only the inverse square law field has the property that its total flux (except at the origin) is conserved. 2.

(b) A conducting sphere of radius R has two non-overlapping cavities of radius $R/4$. It is connected to a voltage source V. A charge $+Q$ is now placed at the centre of one of the cavities. The potential in the other cavity will be unchanged. 2.

(c)There is a nonzero bound surface charge density at the boundary S between the dielectric and vacuum in both the half filled infinite parallel plate capacitors, shown below. 2.

PLEASE RETURN THE FORMULA SHEET AFTER THE MINOR.

$$\epsilon = \epsilon_0(1+\chi_e) = \epsilon_0\epsilon_r$$